***General Mathematical analysis for physical proof***

According to fig. 3, suppose that logic ‘0’ is provided at the input side to cell-1, therefore, cell-2 should follow the same logic as input cell because it is connected in series with cell-1. This judgment requires some kind of a physical proof.

For physical proof, assume that all the cells are similar and length of each one is ‘a’ (a = 18nm) and there is a space of ‘x’ (x = 2nm) between each two neighbor cells [5]. In all the figures, rectangles show the QCA cell and the circles inside it show the position of electrons inside that particular cell. In order to achieve more stability, electrons of QCA cells are arranged in such a manner that their potential energy should be at minimum level.

The potential energy between two electron charges is calculated using relation (1a). In this equation, U is the potential energy, k is fixed colon, q1 and q2 are electric charges and r is the distance between two electric charges. By putting the values of k and q, Eq. (1b) can be obtained. UT is the summation of potential energies that is calculated from Eq. (2). Physical proof has been provided as below:

**(1a)**



**(1b)**

**(2)**

(a) (b)

**Fig. 3** QCA cells connected in series as a Binary wire for (a) logic ‘0’ value in cell-2 and (b) logic ‘1’ value in cell-2

Assumption 1: If cell-2 acquires logic ‘0’ as in fig. 3 (a). Assumption 2: If cell-2 acquires logic ‘1’ as in fig. 3 (b).

Fig. 3 (a) (electron x)

Fig. 3 (a) (electron y)

Fig. 3 (b) (electron x)

Fig. 3 (b) (electron y)

With a comparison to the above result, it can be observed that the potential energy of cell-2 in fig.3 (a) is lower. Therefore, cell-2 will acquire logic ‘0’. Same as, if cell-1 is at logic ‘1’, one can take cell-2 as logic ‘1’ conceding that the two consecutive QCA cells are connected in series with each other. The above mathematical analysis can be applied exhaustively for identifying the general cell positions lying at any place in the direction of each other in QCA structure.

Reference:

[5] Zhang R, Walus K, Wang W, Jullien G (2005) Performance comparison of quantum-dot cellular automata adders Circuits and Systems, IEEE Int. Symp. Circuits Syst. 3: 2522-2526